4 Linear Equations in Two Variables

Fastrack Revision

- **Linear Equation:** An algebraic equation in which the highest degree term is of first degree and whose graph is a straight line.
  For example, $y = 2x + 7$, $x + 2y - z = 4$, etc.
- **Linear Equation in One Variable:** An equation of the form $ax + b = 0$, where a and b are real numbers such that $a \neq 0$ and x is a variable.
  For example, $y = 7y - 4$, $3x - 5 = 0$, etc.
- **Linear Equation in Two Variables:** An equation of the form $ax + by + c = 0$, where a, b and c are real numbers such that a and b are not both zero *i.e.*, $a \neq 0$, $b = 0$ or $a = 0$, $b \neq 0$ or $a \neq 0$, $b \neq 0$ and x, y are variables. For example, $2x + 5y + 7 = 0$, $\sqrt{3}y - \sqrt{3}z = 7$, etc.
- **Solutions of Linear Equation:** The values of variables involved in linear equation which satisfy it *i.e.*, both sides of equation are equal.

Knowledge BOOSTER

1. *A linear equation in one variable has a unique solution.*
2. *A linear equation in two variables has infinitely many solutions.*
3. *The solution of a linear equation is not affected when: (i) the same number is added to (or subtracted from) both the sides of the equation. (ii) the same non-zero number is multiplied or divided on both the sides of the equation.*
4. *An easy way of getting a solution is to take $x = 0$ and get the corresponding value of y. Similarly, we can put $y = 0$ and obtain the corresponding value of x.*

Practice Exercise

Multiple Choice Questions

Q 1. The equation of the Y-axis is:
a. $x = 0$ b. $x = y$
c. $y = 0$ d. $x + y = 0$

Q 2. The equation $2x + 5y = 7$ has a unique solution, if x, y are:
a. natural numbers
b. not real numbers
c. irrational numbers
d. negative real numbers

Q 3. The linear equation $5x + 3y = 14$ has:
a. one solution
b. two solutions
c. three solutions
d. more than three solutions

Q 4. Which of the following line is perpendicular to X-axis?
a. $x + y = 2$ b. $2y + 5 = 0$
c. $3x + 1 = 0$ d. $2x = 5y$

Q 5. The solution of the form $(b, -b)$ always satisfy the equation:
a. $x = -b$ b. $y = -a$
c. $y = x$ d. $x + y = 0$

Q 6. Any point on the line $y = 5x$ is of the form:
a. $\left(a, \frac{a}{5}\right)$ b. $(a, 5a)$
c. $\left(-a, \frac{a}{5}\right)$ d. $\left(a, -\frac{a}{5}\right)$

Q 7. If $x = -2$ and $y = 3$ is a solution of the equation $5mx - 2y = 10$, then the value of m is:
a. $\frac{5}{8}$ b. $\frac{8}{5}$ c. $-\frac{8}{5}$ d. $-\frac{5}{8}$

Q 8. Which of the following ordered pair is a solution of the equation $3x - y - 6 = 0$?
a. $(1, 3)$ b. $(1, -3)$
c. $(-1, 3)$ d. $(-1, -3)$

Q 9. The solution of linear equation $3x + 0y - 6 = 0$ is:
a. $(-2, k)$ b. $(2, k)$
c. $(k, 2)$ d. $(k, -2)$

Q 10. If we multiply or divide both sides of a linear equation with the same non-zero number, then the solution of the linear equation:
a. changes b. remains the same
c. Both a. and b. d. is not defined

Q 11. The positive solutions of the equation $ax + by + c = 0$ always lie in the:

a. 4th quadrant b. 3rd quadrant
c. 2nd quadrant d. 1st quadrant

Q 12. How many linear equations in x and y can be satisfied by $x = 2$ and $y = -3$?

a. Only one b. Two
c. Three d. Infinitely many

Q 13. $x = 5$, $y = 2$ is a solution of the linear equation:

a. $x + 2y = 7$ b. $5x + 2y = 7$
c. $x + y = 7$ d. $5x + y = 7$

Q 14. The coordinates of a point satisfy the equation $x + 3y = 10$ are:

a. (3, 1) b. (1, 3) c. (1, – 3) d. (3, –1)

Q 15. At how many points, the line $3x + 4y = 6$ intersects the X-axis?

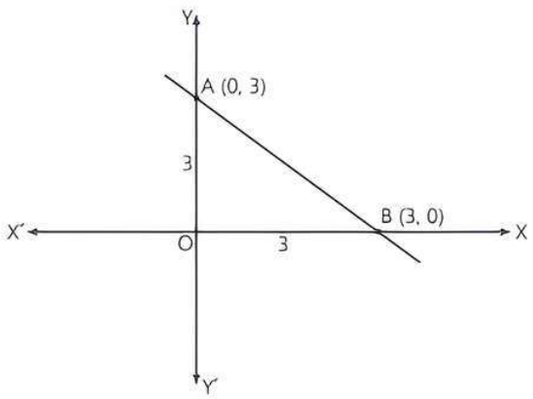
a. 0 b. 1 c. 2 d. 3

Q 16. The linear equation $3x + 6y = 5$ cuts the Y-axis at the point:

a. $\left(-\frac{5}{6}, 0\right)$ b. $\left(0, -\frac{5}{6}\right)$

c. $\left(\frac{5}{6}, 0\right)$ d. $\left(0, \frac{5}{6}\right)$

Q 17. The area of triangle formed by the line $x + y = 3$ and between axes is:



a. 4.5 sq. units b. 5 sq. units
c. 4 sq. units d. 5.3 sq. units

Assertion & Reason Type Questions

Directions (Q. Nos. 18-22): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 18. Assertion (A): If $x = -1$ and $y = 2$ is a solution of the equation $3x + 2y = k$, then the value of k is 1.

Reason (R): The solution of the line will satisfy the equation of the line.

Q 19. Assertion (A): The linear equation $2x + 3y = 5$ has a unique solution.

Reason (R): A linear equation in two variables has infinitely many solutions.

Q 20. Assertion (A): The point (2, 1) satisfy the linear equation $3x + 4y = 10$.

Reason (R): Any point satisfy the linear equation means when we put the coordinate values in the given equation, then LHS = RHS.

Q 21. Assertion (A): $y = 3$ is a line, which is 3 units distance from X-axis.

Reason (R): $X = h$ is a line, which is h unit distance from Y-axis.

Q 22. Assertion (A): The linear equation $3x - 2y = 4$ passes through the point (3, 4).

Reason (R): Every point lying on the line is a solution of the equation $3x - 2y = 4$.

Fill in the Blanks Type Questions

Q 23. A linear equation in one variable has a ..... solution.

Q 24. The value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$, is ..... .

Q 25. The point of the form (a, a) always satisfy the equation ..... .

Q 26. If $(-2, 3)$ is the solution of the linear equation $2x + y - a = 0$, then the value of a is ..... .

Q 27. The linear equation $2x + y = 8$ cuts the X-axis at the point ..... .

True/False Type Questions

Q 28. If the cost of 2 cricket bat and 3 balls is ₹240, the expression in the form of linear equation is $2x + 3y = 240$.

Q 29. (0, 2) is a solution of the linear equation $x - 2y = 4$.

Q 30. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, then the value of a is $\frac{2}{3}$.

Q 31. The line of the linear equation $7x - y = 0$ passes through the point (2, 14).

Q 32. The equation $2x - \frac{3}{2}y = 8$ passes through the origin.

Solutions

1. (a) $x = 0$

2. (a) The equation $2x + 5y = 7$ has a unique solution if x, y are natural numbers.

3. (d) More than three solutions.

4. (c) In option:

(a) $x + y = 2$ is a line which intercept both the axes at points (2, 0) and (0, 2).

(b) $2y + 5 = 0$ or $y = -\frac{5}{2}$ is a line which is perpendicular to Y-axis at a distance $\frac{5}{2}$ units below the X-axis.

(c) $3x + 1 = 0$ or $x = -\frac{1}{3}$ is a line which is perpendicular to X-axis at a distance $\frac{1}{3}$ from left side of Y-axis.

5. (d) At point $(b, -b)$:

(a) equation $x = -b$ at given point is $b = -b$, which is not correct.

(b) equation $y = -a$ at given point is $-b = -a$, which is not correct.

(c) equation $y = x$ at given point is $b = -b$, which is not correct.

(d) equation $x + y = 0$ at given point is $b - b = 0 \Rightarrow 0 = 0$, which is correct.

6. (b) Given equation is

$$y = 5x$$

At point $\left(a, \frac{a}{5}\right)$,

$$\frac{a}{5} = 5a, \text{ which is not correct.}$$

At point $(a, 5a)$, $5a = 5a$, which is correct.

7. (c) Given equation is $5mx - 2y = 10$.

Put $x = -2$ and $y = 3$ in the given equation.

Therefore $5m(-2) - 2(3) = 10$

$\Rightarrow -10m - 6 = 10$

$\Rightarrow -10m = 16$

$\Rightarrow m = -\frac{16}{10}$

$\Rightarrow m = -\frac{8}{5}$

8. (b) Given equation is $3x - y - 6 = 0$.

At point (1, 3),

$3(1) - 3 - 6 = 3 - 3 - 6$

$= -6 \neq 0$, which is not correct.

At point $(1, -3)$,

$3(1) - (-3) - 6 = 3 + 3 - 6$

$= 0$, which is correct.

9. (b) Given equation is $3x + 0y - 6 = 0$

$\Rightarrow 3x = 6 \Rightarrow x = 2 \Rightarrow y = k$

where k is any real number.

Hence, solution of given equation is $(2, k)$.

10. (b) If we multiply or divide both sides of a linear equation with the same non-zero number, then the solution of the linear equation remains the same.

11. (d) The positive solutions of the given equation always lie in 1st quadrant.

12. (d) The number of linear equations in x and y can be satisfied by $x = 2$ and $y = -3$ are infinitely many.

13. (c) When $x = 5$, $y = 2$, then

(a) equation $x + 2y = 7$ has

$5 + 2(2) = 5 + 4 = 9 \neq 7$, no solution

(b) equation $5x + 2y = 7$ has

$5(5) + 2(2) = 25 + 4$

$= 29 \neq 7$, no solution

(c) equation $x + y = 7$ has

$5 + 2 = 7 =$ RHS, solution

14. (b) Given equation is $x + 3y = 10$.

(a) At point (3, 1), equation

$3 + 3(1) = 6 \neq 10$ is not satisfy

(b) At point (1, 3), equation is

$1 + 3(3) = 1 + 9$

$= 10 =$ RHS, is satisfy

Hence, coordinates of point (1, 3) satisfy the given equation.

15. (b) The line $3x + 4y = 6$ intersects the X-axis, when $y = 0$. Then

$3x + 4(0) = 6 \Rightarrow x = \frac{6}{3}$

$\Rightarrow x = 2$

Hence, point (2, 0) is the only point which intersect the X-axis.

16. (d) The linear equation $3x + 6y = 5$ cuts the Y-axis, when $x = 0$. Then

$3(0) + 6y = 5$

$\Rightarrow y = \frac{5}{6}$

Hence, given linear equation cuts the Y-axis at point $\left(0, \frac{5}{6}\right)$.

17. (a) Given line intersect the coordinate axes at points A(0, 3) and B(3, 0).

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 3 \times 3$

$= \frac{9}{2} = 4.5$ sq. units

18. (a) **Assertion (A):** Given equation is $3x + 2y = k$.

Since, $x = -1$ and $y = 2$ is a solution of $3x + 2y = k$.

Therefore $3(-1) + 2(2) = k$

$\Rightarrow \quad -3 + 4 = k \quad \Rightarrow \quad k = 1$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

19. (d) **Assertion (A):** A linear equation $2x + 3y = 5$ has not a unique solution.

So, Assertion (A) is false.

Reason (R): It is true to say that the linear equation in two variables has infinitely many solutions.

Hence, Assertion (A) is false but Reason (R) is true.

20. (a) **Assertion (A):** Since, Point (2, 1) satisfy the linear equation $3x + 4y = 10$.

Put $x = 2$ and $y = 1$ in the left hand side of the given equation.

$\therefore \quad LHS = 3(2) + 4(1) = 6 + 4$

$= 10 = RHS$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21. (b) **Assertion (A):** It is true that, $y = 3$ is a line, which is 3 units distance from X-axis in the positive direction of Y-axis.

Reason (R): It is true that $X = h$ is a line, which is h unit distance from Y-axis.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

22. (d) **Assertion (A):** Put $x = 3$ and $y = 4$ in the left hand side of the given equation.

$\therefore \quad LHS = 3(3) - 2(4) = 9 - 8$

$= 1 \neq RHS$

So, Assertion (A) is false.

Reason (R): It is true to say that every point lying on the line is a solution of the equation

$3x - 2y = 4.$

Hence, Assertion (A) is false but Reason (R) is true.

23. unique

24. Given equation is $2x + 3y = k$.

Since, $x = 2$ and $y = 1$ is the solution of the given equation. Therefore put $x = 2$ and $y = 1$ in the given equation, we get

$2(2) + 3(1) = k$

$\Rightarrow \quad k = 4 + 3$

$\Rightarrow \quad k = 7$

25. The equation having both coordinates are same *i.e.*, (a, a) is in the form of $y = x$.

26. Since, $(-2, 3)$ is the solution of $2x + y - a = 0$.

Therefore, put $x = -2$ and $y = 3$ in the given equation, we get

$2(-2) + 3 - a = 0$

$\Rightarrow \quad -4 + 3 - a = 0$

$\Rightarrow \quad a = -1$

27.

TiP

If any linear equation cuts the X-axis, then its y-coordinate should be zero.

The linear equation $2x + y = 8$ cuts the X-axis, then put $y = 0$, we get

$2x + 0 = 8$

$\Rightarrow \quad x = 4$

Hence, coordinates of that point on X-axis is $(4, 0)$.

28. True

29. False

Given linear equation is

$x - 2y = 4$

Put $x = 0$ and $y = 2$ in LHS, we get

$LHS = 0 - 2(2)$

$= -4 \neq RHS$

Hence, given statement is false.

30. False

Since, the point (3, 4) lies on the graph of the equation $3y = ax + 7$. Therefore put $x = 3$ and $y = 4$ in the given equation, we get

$3(4) = 3a + 7$

$\Rightarrow \quad 3a = 12 - 7$

$\Rightarrow \quad 3a = 5$

$\Rightarrow \quad a = \frac{5}{3}$

Hence, given statement is false.

31. True

Given linear equation is

$7x - y = 0$

At point (2, 14),

$7(2) - 14 = 0$

$\Rightarrow \quad 0 = 0$

So, the given linear equation passes through the point (2, 14).

Hence, given statement is true.

32. False

Given equation is $2x - \frac{3}{2}y = 8$

At point (0, 0),

$2(0) - \frac{3}{2}(0) = 8$

$\Rightarrow \quad 0 = 8$, which is not correct.

Hence, given statement is false.

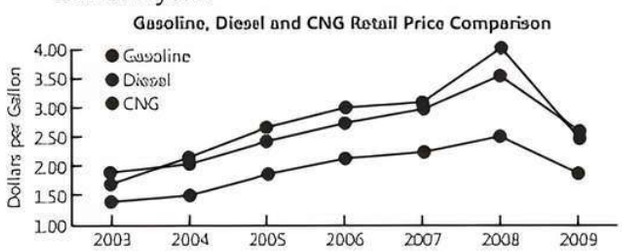
Case Study Based Questions

Case Study 1

As petrol, diesel and CNG are essential component of manufacturing sector and transportation. Due to increase in price fuel, the price of all consumers product directly increases. Suppose, there are two statements which are based on petrol and CNG that are given below:

Statement-I: Every one month price of petrol increases at the rate of ₹ 2 per litre. Consider the price of petrol in month of January 2020 as ₹ x and present price of petrol as ₹ y.

Statement-II: Because of continuous increase in the price of petrol, people found other option as CNG, whose price increases at the rate of ₹ 4 per litre in a year.



On the basis of the above information, solve the following questions:

Statement I

Q 1. By using statement-I, form a linear equation that the amount spend on petrol in beginning of January 2021.

a. $x = y + 24$ b. $y = x + 24$
c. $y = x + 2$ d. $y = x - 24$

Q 2. If $x = 5$, then the value of y is:

a. 28 b. 25
c. 29 d. 30

Statement II

Q 3. By using statement-II, form a linear equation taking price of CNG in January 2020 as l and its going upto January 2021 as:

a. $l = m + 4$ b. $2l = m + 4$
c. $m = l + 4$ d. $2m = l + 4$

Q 4. If $m = 2$, then the value of l is:

a. -4 b. -2
c. 2 d. 3

Q 5. Which of the following point (l, m) satisfy the equation $m = 2l + 4$?

a. (2, 0) b. (– 1, 3)
c. (1, 6) d. $\left(\frac{1}{2}, -4\right)$

Solutions

1. (b) It is given that price of petrol increase every month at the rate of ₹ 2 per litre. Therefore in 12 months, total price increase of
₹ 12×2 *i.e.* ₹ 24.
∴ The linear equation will be formed as
$y = x + 24$
So, option (b) is correct.
2. (c) We have $y = x + 24$
When $x = 5$, then
$y = 5 + 24 = 29$
So, option (c) is correct.
3. (c) The required linear equation formed by statement-II is $m = l + 4$
So, option (c) is correct.
4. (b) We have, $m = l + 4$
When $m = 2$, then
$2 = l + 4 \Rightarrow l = -2$
So, option (b) is correct.
5. (c) Given equation is
$m = 2l + 4$
(a) At point (2, 0),
$0 = 2 \times 2 + 4 \Rightarrow 0 = 8$,
Which is not true.
(b) At point (– 1, 3),
$3 = 2(-1) + 4 \Rightarrow 3 = 2$,
Which is not true
(c) At point (1, 6),
$6 = 2(1) + 4 \Rightarrow 6 = 6$,
Which is true.
Hence, point (1, 6) satisfy the given equation.
So, option (c) is correct.

Case Study 2

Vehicle parking is the major problem in any metropolitan city. In Delhi at Chandni Chowk, the parking charge of a two wheeler is as ₹ 20 for the first two hours and ₹ 5 for next subsequent hours. Suppose total charge of a two wheeler is ₹ x and total parking time is y hours.

On the basis of the above information, solve the following questions:

Q 1. Write a linear equation in the given statement:

a. $5 + 5x = y$ b. $5x = y + 10$
c. $10 + 5x = y$ d. $10 - 5x = y$

Q 2. If $x = -1$, then the value of y is:

a. 3 b. 5
c. – 5 d. – 3

Q 3. If $y = 20$, then the value of x is:
a. 4 b. -2 c. 3 d. 2

Q 4. Which of the following point satisfy the equation $10 + 5x = y$?
a. $(-1, 3)$ b. $(1, 15)$ c. $(2, 15)$ d. $(-2, 1)$

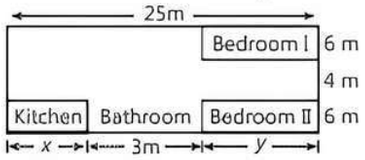
Q 5. A linear equation $10 + 5x = y$ satisfy:
a. only one point b. atmost two points
c. infinitely points d. zero point

Solutions

1. (c) It is given that charges of first two hours is ₹ 20 and the charges of next subsequent hour is ₹ 5 *i.e.* for $(x - 2)$ hours, charges for per hour is ₹ 5.
According to the given condition,
$$20 + 5(x - 2) = y$$
$$\Rightarrow \quad 20 + 5x - 10 = y$$
$$\Rightarrow \quad 10 + 5x = y$$
So, option (c) is correct.
2. (b) We have, $10 + 5x = y$
When $x = -1$, then
$$10 + 5(-1) = y$$
$$\Rightarrow \quad y = 10 - 5 = 5$$
So, option (b) is correct.
3. (d) We have $10 + 5x = y$, then
Put $y = 20$, we get
$$10 + 5x = 20$$
$$\Rightarrow \quad 5x = 10$$
$$\Rightarrow \quad x = 2$$
So, option (d) is correct.
4. (b) We have, $10 + 5x = y$
(a) At point $(-1, 3)$,
$$10 + 5(-1) = 3$$
$$\Rightarrow \quad 5 = 3,$$
which is not true.
(b) At point (1, 15),
$10 + 5(1) = 15 \Rightarrow 15 = 15$, which is true.
Hence, point (1, 15) satisfy the equation
$$10 + 5x = y.$$
So, option (b) is correct.
5. (c) A linear equation $10 + 5x = y$ satisfy infinitely many points.
So, option (c) is correct.

Case Study 3

Gupta's family wanted to purchase a house near national highway 54. One day, they went to the property dealer and saw the different maps of houses there. One of the map was shown below.



On the basis of the above information, solve the following questions:

Q 1. Find the area of one kitchen and one bedroom.

Q 2. Write the linear equation in two variables formed by the given layout.

Q 3. Find the number of solutions exist in the equation $x + y = 22$.

Solutions

1. From figure, length of kitchen = x m
and width of kitchen = 6 m
$\therefore$ Area of kitchen = length × width
$$= x \times 6 = 6x \text{ m}^2$$
From figure, length of bedroom = y m
and width of bedroom = 6 m
$\therefore$ Area of one bedroom = length × width
$$= y \times 6 = 6y \text{ m}^2.$$
Hence, area of kitchen is $6x$ m^2 and area of one bedroom is $6y$ m^2.
2. From given layout,
$$x + 3 + y = 25$$
$$\Rightarrow \quad x + y = 22$$
3. There are infinitely many solutions exist in the equation $x + y = 22$.

Case Study 4

Sumit went to the market and ask the fruit seller about the rates of different fruits. He said that the cost of 3 kg apples and 2 kg of guava on a particular day was found to be ₹ 200. On the next day, the cost of 6 kg of apples and 3 kg of guava is ₹ 360.

Suppose x and y represent the quantity of apples and guava.

On the basis of the above information, solve the following questions:

Q 1. Find the algebraic representation of both conditions.

Q 2. By using II condition, find the value of x, when $y = 10$.

Q 3. By using I condition, if Sumit purchase an apple of ₹ 40, then find the amount required to purchase guava.

Solutions

1. The algebraic representation of both conditions are
$$3(x) + 2(y) = 200$$
$$\Rightarrow \quad 3x + 2y = 200$$
and $\quad 6(x) + 3(y) = 360$
$$\Rightarrow \quad 6x + 3y = 360 \quad \text{or} \quad 2x + y = 120$$
2. From II condition, $2x + y = 120$
When $y = 10$, then
$$2x + 10 = 120$$
$$\Rightarrow \quad 2y = 110$$
$$\Rightarrow \quad x = 55$$

3. From I condition,

$$3x + 2y = 200$$

When $x = 40$, then

$$3(40) + 2y = 200$$

$$\Rightarrow \quad 2y = 200 - 120$$

$$\Rightarrow \quad 2y = 80$$

$$\Rightarrow \quad y = 40$$

Hence, Sumit required ₹ 40 to purchase guava.

Very Short Answer Type Questions

Q 1. Is $0x + 0y + c = 0$, a linear equation?

Q 2. Express $\frac{x}{4} - 3y = 7$ in the form of $ax + by + c = 0$.

Q 3. In a one day cricket match, A and B scored 125 runs. Express this as linear equation in two variables.

Q 4. Does the equation $x = 5y$ represent a straight line passing through the point (0, 0)?

Q 5. Write the given equation $x = 7$ in two variables.

Q 6. At $x = 1$, find the coordinates of the equation $3x + 2y = 12$.

Q 7. Write one solution of $\pi x + y = 5$.

Q 8. If the point (2, 3) lies on the line $4y = ax + 5$, then find the value of a.

Q 9. Find the value of k for which $(2, -3)$ is a solution of $2x + y = k$.

Q 10. If the line $2x + ky = 5$ passes through the point $(-2, 1)$, then find k.

Q 11. Where does the point of the form $(-a, -a)$ always lie?

Short Answer Type-I Questions

Q 1. Write any two solutions of the linear equation $2x + 5y = 10$.

Q 2. The cost of a pencil is ₹ 5 less than half of the cost of a ball pen. Write this statement as a linear equation in two variables.

Q 3. Check which of the following is (are) the solution(s) of the equation $3y - 2x = 1$

(i) $(4, 3)$ (ii) $(2\sqrt{2}, 3\sqrt{2})$

Q 4. For what value of c, the linear equation $2x + cy = 8$, has equal values of x and y for its solution?

Q 5. Let y varies directly as x. When $y = 10$ and $x = 2$, then write a linear equation. What is value of y when $x = 6$?

Q 6. If the point $(2k - 3, k + 2)$ lies on the equation $2x + 3y + 15 = 0$, find the value of k.

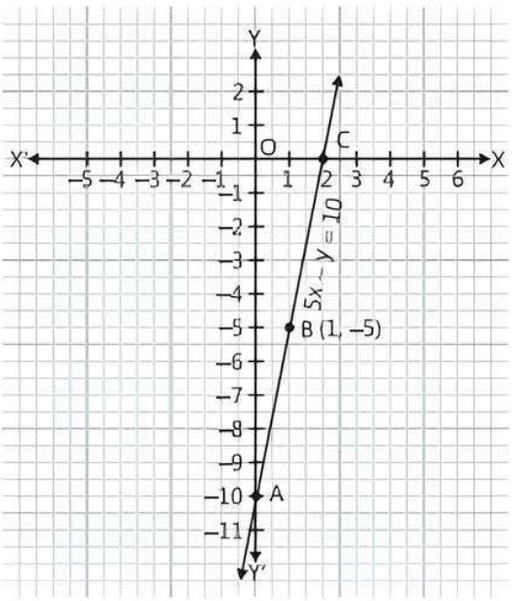
Q 7. Find the value of k, if $(2, -2)$ is a solution of the equation $4x - ky = 6$. Also, find the coordinates of another point lying on its equation.

Short Answer Type-II Questions

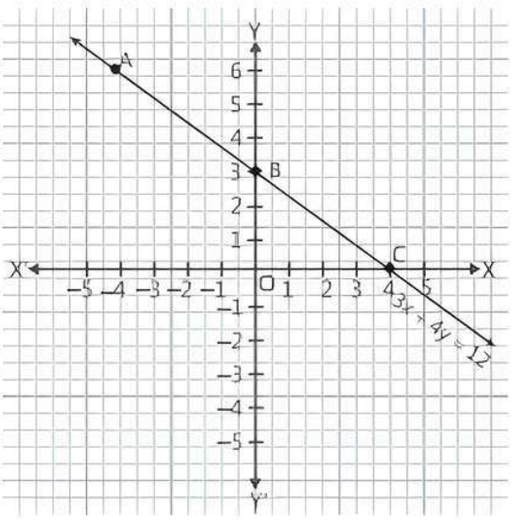
Q 1. Write the equation $y\sqrt{3} = 8x + \sqrt{3}$ in the form of $ax + by + c = 0$. Check whether $(0, -1)$ and $(\sqrt{3}, 9)$ are the solutions of this equation.

Q 2. The cost of a toy elephant is the same as the cost of 3 balls. Express the statement as a linear equation in two variables. Write any two solutions of the equation.

Q 3. Find the coordinates of A and C. Also, find the area of AOC.



Q 4. Find the coordinates of B and C. Check whether P (2, 1.5) and Q (–3, 4) are the solution of the given equation or not. Find the length BC.



Q 5. Find the coordinates of the points where the line representing the equation $\frac{x}{4}=1-\frac{y}{6}$ cut the X-axis and Y-axis. Also, find the distance between two intersecting points on coordinate axes.

Q 6. For what value of p; $x = 2$, $y = 3$ is solution of $(p+1)\,x-(2p+3)\,y-1=0$ and write the equation?

Q 7. Give the equations of two lines passing through the point (2, 14). How many more such lines are there and why?

Long Answer Type Questions

Q 1. The taxi fare in a city is as follows. For the first kilometre, the fare is ₹ 20 and for the subsequent distance, it is ₹ 10 per km. Taking distance covered as x km and the total fare as ₹ y, write a linear equation for this information and draw its graph. Using graph, find the fare for: (i) 10 km, (ii) 12 km.

Q 2. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in celsius. Here is a linear equation that converts Fahrenheit to celsius.

$$F=\left(\frac{9}{5}\right)C+32$$

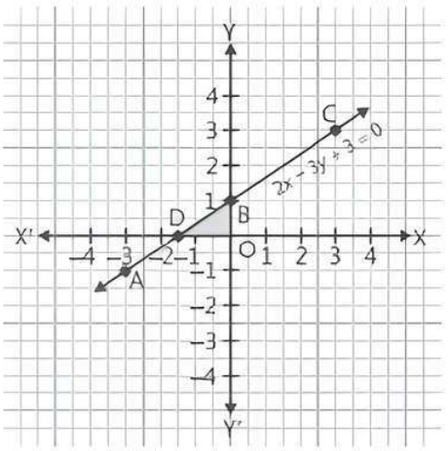
(i) If the temperature is 30°C, what is the temperature in Fahrenheit?

(ii) If the temperature is 95°F, what is the temperature is celsius?

(iii) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in celsius?

(iv) Is there a temperature which is numerically the same in both Fahrenheit and celsius? If yes, find it.

Q 3. The graph of linear equation $2(x+3)-3(y+1)=0$, is shown below. Answer the following questions:

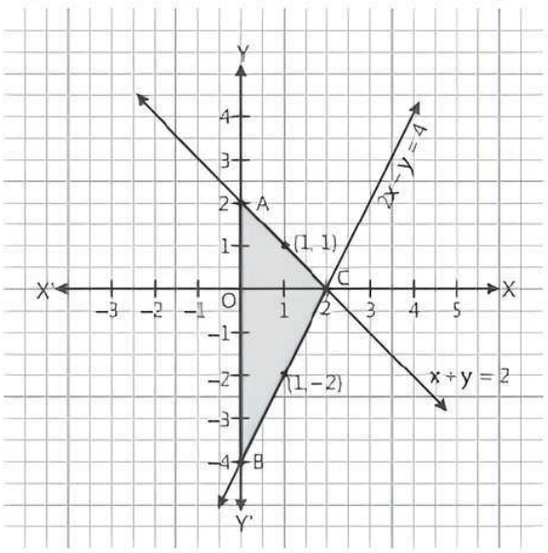


(i) Write the quadrants in which the line segment intercepted between the axes lie.

(ii) Write the vertices of the triangle so formed.

(iii) Find the area of triangle so formed.

Q 4. The graphs of $2x-y=4$, $x+y=2$ is shown below. Write the coordinates of vertices of the triangle. Also, find the area and perimeter of a triangle.



Solutions

Very Short Answer Type Questions

1. No, because for a linear equation $ax+by+c=0$, either $a\neq 0$ or $b\neq 0$.
2. Given equation is $\frac{x}{4}-3y=7$.
$$\Rightarrow \quad \frac{x-12y}{4}=7$$
$$\Rightarrow \quad x-12y=28$$
$$\Rightarrow \quad x-12y-28=0$$
3. Let the runs scored by A and B be x and y respectively, then
$$x+y=125$$
4. Yes, Given equation is $x = 5y$.
The point (0, 0) satisfies the equation, so it is a straight line.
5. $1\cdot x+0\cdot y=7$
6. At $x = 1$,
$3(1)+2(y)=12 \Rightarrow 2y=9$
$\Rightarrow y=4.5$
Hence, required coordinate is (1, 4.5).
7. Given equation is $\pi x+y=5$
$\Rightarrow y=5-\pi x$
Let $x = 1$, then $y=5-\pi\times 1=5-\pi$
Hence, $(1, 5-\pi)$ is one solution of $\pi x+y=5$.

8. Given, point (2, 3) lies on the line $4y = ax + 5$.

$\therefore \quad 4(3) = a(2) + 5$

$\Rightarrow \quad 2a = 12 - 5 = 7$

$\Rightarrow \quad a = \frac{7}{2}$

9. Given equation is $2x + y = k$.

On putting $x = 2, y = -3$, we get

$2 \times 2 + (-3) = k$

$\Rightarrow \quad 4 - 3 = k$

$\Rightarrow \quad k = 1$

Hence, the value of k is 1.

10. Given line $2x + ky = 5$ passes through the point $(-2, 1)$.

$\therefore \quad 2(-2) + k(1) = 5$

$\Rightarrow \quad -4 + k = 5$

$\Rightarrow \quad k = 9$

11. The point of the form $(-a, -a)$ must always lies on the line $y = x$ because such a point always satisfy the equation of the line *i.e.*, $y = x$.

Short Answer Type-I Questions

1. Given equation is $2x + 5y = 10$.

$\Rightarrow \quad y = \frac{10 - 2x}{5}$

When $x = 0$, then

$y = \frac{10 - 2 \times 0}{5} = \frac{10 - 0}{5} = 2$

When $x = 5$, then

$y = \frac{10 - 2 \times 5}{5} = \frac{10 - 10}{5} = \frac{0}{5} = 0$

Hence, (0, 2) and (5, 0) are the two solutions of the given equation.

2. Let the cost of a pencil = ₹ x

and the cost of a ball pen = ₹ y

According to the given condition,

$x = \frac{y}{2} - 5$

$\Rightarrow \quad x = \frac{y - 10}{2}$

$\Rightarrow \quad 2x = y - 10 \quad \Rightarrow \quad 2x - y + 10 = 0$

Hence, the required linear equation is

$2x - y + 10 = 0$

3. Given equation is $3y - 2x = 1$

(i) When we put $x = 4, y = 3$

Then equation $3y - 2x = 1 \Rightarrow 3(3) - 2(4) = 1$

$\Rightarrow \quad 1 = 1$

$\therefore$ (4, 3) is the solution of the equation.

(ii) When we put $x = 2\sqrt{2}, y = 3\sqrt{2}$

Then equation $3y - 2x = 1$

$\Rightarrow \quad 3(3\sqrt{2}) - 2(2\sqrt{2}) = 1$

$\Rightarrow \quad 5\sqrt{2} \neq 1$

$\therefore \quad (2\sqrt{2}, 3\sqrt{2})$ is not the solution of the given equation.

4. Given equation is $2x + cy = 8$. ...(1)

According to the given condition, $x = y$

On putting $x = y$ in eq. (1), we get

$2x + cx = 8 \Rightarrow cx = 8 - 2x$

$\Rightarrow \quad c = \frac{8 - 2x}{x}, x \neq 0$

Hence, the required value of c is $\frac{8 - 2x}{x}$.

5. Given that y varies directly as x.

i.e., $y \propto x \Rightarrow y = kx$ ...(1)

where k is arbitrary constant.

Given, $y = 10$ and $x = 2$

$\therefore \quad 10 = 2k \Rightarrow k = 5$

Put $k = 5$ in eq. (1), we get

$y = 5x$.

When $x = 6$, then

$y = 5 \times 6$

$\Rightarrow \quad y = 30$

6. Given equation is $2x + 3y + 15 = 0$. ...(1)

Since, point $(2k - 3, k + 2)$ lies on the given equation, so it must satisfies it.

On putting $x = 2k - 3$ and $y = k + 2$ in eq. (1), we get

$2(2k - 3) + 3(k + 2) + 15 = 0$

$\Rightarrow \quad 4k - 6 + 3k + 6 + 15 = 0$

$\Rightarrow \quad 7k + 15 = 0$

$\Rightarrow \quad 7k = -15 \Rightarrow k = -\frac{15}{7}$

Hence, the value of k is $-\frac{15}{7}$.

7. Given, point $(2, -2)$ is a solution of the equation

$4x - ky = 6$ ...(1)

Put $x = 2$ and $y = -2$, we get

$4(2) - k(-2) = 6$

$\Rightarrow \quad 8 + 2k = 6 \Rightarrow 2k = -2$

$\Rightarrow \quad k = -1$

Put $k = -1$ in eq. (1), we get

$4x - (-1)y = 6 \Rightarrow 4x + y = 6$

Consider $x = 3$, then

$4(3) + y = 6$

$\Rightarrow \quad y = -6$

Hence $k = -1$ and another point lying on equation is $(3, -6)$.

Short Answer Type-II Questions

1. Given equation is $y\sqrt{3} = 8x + \sqrt{3}$

$\Rightarrow \quad 8x - y\sqrt{3} + \sqrt{3} = 0$

On putting $x = 0, y = -1$, we get

$8(0) - \sqrt{3}(-1) + \sqrt{3} = \sqrt{3} + \sqrt{3} = 2\sqrt{3} \neq 0$

$\therefore$ (0, –1) is not a solution of the given equation.

On putting $x = \sqrt{3}, y = 9$, we get

$8(\sqrt{3}) - \sqrt{3}(9) + \sqrt{3} = 8\sqrt{3} - 8\sqrt{3} = 0$

$\therefore \quad (\sqrt{3}, 9)$ is a solution of the given equation.

2. Let the cost of a toy elephant = ₹ x
and cost of a ball = ₹ y
According to the given condition,

$$3y = x \Rightarrow y = \frac{x}{3}$$

Put $x = 1$, then

$$y = \frac{1}{3}$$

Put $x = 6$, then

$$y = \frac{6}{3} = 2$$

Hence, any two solutions of the equation are $\left(1, \frac{1}{3}\right)$ and (6, 2).

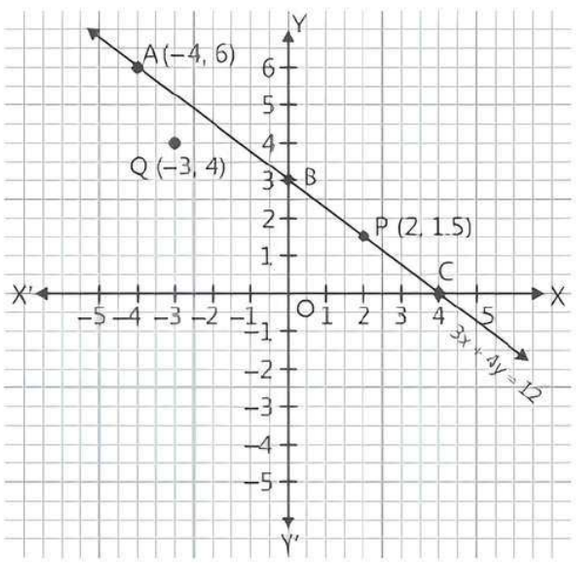
3. The coordinates of the given points are A (0, –10) and C (2, 0).

$$\therefore \text{ Area of } \Delta AOC = \frac{1}{2} \times OC \times OA$$

$$= \frac{1}{2} \times 2 \times 10$$

$$= 10 \text{ sq. units}$$

4.



The coordinates of B and C are (0, 3) and (4, 0).
When we plot the points P and Q on a graph paper, we see that point Q (–3, 4) does not lie on a line but point P (2, 1.5) lies on a line.

Now, length $BC = \sqrt{(OC)^2 + (OB)^2}$

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16 + 9} = \sqrt{25}$$

$$= 5 \text{ units}$$

5. Given equation is $\frac{x}{4} = 1 - \frac{y}{6}$.

$$\Rightarrow \frac{x}{4} + \frac{y}{6} - 1 = 0 \Rightarrow \frac{3x + 2y - 12}{12} = 0$$

$$\Rightarrow 3x + 2y - 12 = 0$$

On X-axis, put $y = 0$

$$\therefore 3x - 12 = 0$$

$$\Rightarrow x = 4$$

$\therefore$ Point on the X-axis is P (4, 0).

On Y-axis, put $x = 0$

$$\therefore 2y - 12 = 0 \Rightarrow y = 6$$

$\therefore$ Point on the Y-axis is Q (0, 6).

$\therefore$ The distance between two intersecting points on coordinate axes is

$$PQ = \sqrt{(4)^2 + (6)^2}$$

$$= \sqrt{16 + 36} = \sqrt{52}$$

$$= 2\sqrt{13} \text{ units}$$

6. Given equation is $(p + 1)x - (2p + 3)y - 1 = 0$ ...(1)

$\because$ $x = 2, y = 3$ is a solution of the given equation.
So, on putting the values of 'x' and 'y' in eq. (1), we get

$$(p + 1)\,2 - (2p + 3)3 - 1 = 0$$

$$\Rightarrow 2p + 2 - 6p - 9 - 1 = 0$$

$$\Rightarrow -4p - 8 = 0$$

$$\Rightarrow p = -2$$

Now, on putting the value of 'p' in eq. (1), we get

$$(-2 + 1)x - [2(-2) + 3]y - 1 = 0$$

$$\Rightarrow -x + y - 1 = 0$$

Hence, $x - y + 1 = 0$ is the required equation.

7. Let $x + y = k$ be such a line that passes through the point (2, 14).

$$\therefore 2 + 14 = k \Rightarrow k = 16$$

Hence, $x + y = 16$ passes through the point (2, 14).
Let $2x + 3y = k'$ be another line that passes through the point (2, 14).

$$\therefore 2 \times 2 + 3 \times 14 = k'$$

$$\Rightarrow k' = 4 + 42 = 46$$

Hence, $2x + 3y = 46$ passes through the point (2, 14).
So, there are infinitely many such equations, which can satisfy a given point.

Long Answer Type Questions

1. Given, fare for the first kilometre = ₹ 20
Fare for the subsequent distance
= ₹ 10 per km
So, fare for the next $(x - 1)$ km = $(x - 1) \times 10$
[$\because$ Total distance = x km]
According to the given condition,
Total fare = Fare for first kilometre
+ Fare for $(x - 1)$ km

$$\therefore y = ₹\,20 + ₹\,(x - 1) \times 10 \quad [\because \text{Total fare} = ₹\,y]$$

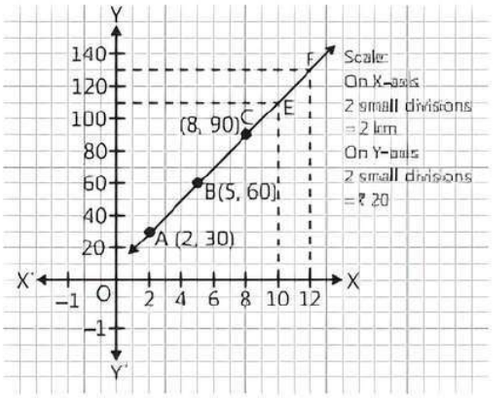
$$= ₹\,(20 + 10x - 10) = ₹\,(10 + 10x)$$

$$\Rightarrow y = 10 + 10x$$

Table of the line $y = 10 + 10x$:

x	2	5	8
y	30	60	90

Plot the points A (2, 30), B (5, 60) and C (8, 90) in the cartesian plane and join them.



The line AC is the required graph.

(i) To find fare for 10 km, firstly draw a line perpendicular to X-axis at point $x = 10$, which intersect the line AB at E and this point, draw a line perpendicular to Y-axis. It meets Y-axis at 110.

Hence, fare for 10 km is ₹ 110.

(ii) Similarly, fare for 12 km is ₹ 130.

2. Fahrenheit-Celsius Temperature Conversion equation

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Value of 30°C Temperature in Fahrenheit

Given equation $F = \left(\frac{9}{5}\right)C + 32$

Put $C = 30$

$$F = \left(\frac{9}{5} \times 30°\right) + 32 = 54 + 32 = 86$$

Therefore, 30°C = 86°F

(ii) Value of 95° F temperature in Fahrenheit.

Given equation, $F = \left(\frac{9}{5}\right)C + 32$

Put $F = 95$

$$95 = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = 95 - 32 = 63$$

$$\therefore \quad C = \frac{63 \times 5}{9} = 35$$

Therefore 95° F = 35° C.

(iii) Fahrenheit temperature of 0°C temperature and celsius temperature of 0°F temperature.

Given equation $F = \left(\frac{9}{5}\right)C + 32$

Put $C = 0$, $F = \left(\frac{9}{5}\right) \times 0 + 32 = 32$

Put $F = 0$, $0 = \frac{9}{5}C + 32$

$$\therefore \quad \frac{9}{5}C = -32$$

$$\therefore \quad C = -\frac{32 \times 5}{9} = \frac{-160}{9} = -17.77$$

Therefore, 0°C = 32°F and 0°F = – 17.77°C.

(iv) Numerically equal reading on both scales

Let the equal number be x

Given equation is $F = \left(\frac{9}{5}\right)C + 32$

Substituting x in place of F and C,

$$x = \frac{9}{5}x + 32$$

$$x - \frac{9}{5}x = 32$$

$$-\frac{4}{5}x = 32$$

$$\therefore \quad x = -\frac{32 \times 5}{4} = -40°$$

Hence, at – 40° both scales will be same reading.

3. (i) The line segment intercepted between the axes lies in II quadrant.

(ii) Vertices of ΔOBD are O(0, 0), B(0, 1) and D(–1.5, 0).

(iii) Area of $\Delta OBD = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$= \frac{1}{2} \times OD \times OB = \frac{1}{2} \times 1.5 \times 1$$

$$= 0.75 \text{ sq. units}$$

4. In the given graph, the vertices of ΔABC are A(0, 2), B(0, –4) and C(2, 0).

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 6 \times 2 = 6 \text{ sq. units}$$

Now, in right ΔAOC,

$$AC = \sqrt{(OA)^2 + (OC)^2}$$

$$= \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4}$$

$$= 2\sqrt{2} \text{ sq. units.}$$

And in right ΔBOC,

$$BC = \sqrt{(OC)^2 + (OB)^2}$$

$$= \sqrt{(2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20}$$

$$= 2\sqrt{5} \text{ units.}$$

$\therefore$ Perimeter of a $\Delta ABC = AB + AC + BC$

$$= (6 + 2\sqrt{2} + 2\sqrt{5}) \text{ units}$$

Chapter Test

Multiple Choice Questions

Q 1. $x = 2$ and $y = -1$ is a solution of the linear equation:

a. $x + 2y = 0$ b. $x + 2y = 4$

c. $2x + y = 0$ d. $2x + y = 5$

Q 2. If $x = -4$ and $y = 5$ is a solution of the equation $7lx - 3y = 10$, then the value of l is:

a. $\frac{25}{28}$ b. $-\frac{25}{28}$

c. $-\frac{28}{25}$ d. $\frac{28}{5}$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4) *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. **Assertion (A): A linear equation $3x + 5y = 15$ has a unique solution.**

Reason (R): A linear equation in two variables has infinitely many solutions.

Q 4. **Assertion (A): The linear equation $5x - 2y = 0$ passes through the point (2, 2).**

Reason (R): Every point lying on the graph is a solution of the equation $5x - 2y = 0$.

Fill in the Blanks

Q 5. **The point $(-b, -b)$ always lies on the line having equation as ..... .**

Q 6. **The graph of the linear equation $3x + 6y = 12$ cuts the X-axis at the point ..... .**

True/False

Q 7. **If the cost of 3 cricket bat and 7 balls is ₹ 150, the expression in the form of linear equation is**

$$3x + 7y = 150$$

Q 8. **The linear equation $3x - \frac{5}{2}y = 4$ passes through the point (3, 2).**

Case Study Based Question

Q 9. One renowned cell phone company has policy to make maximum number of users. So, he offered two different packages which are given below:

I Package: In this package, company charges a fixed monthly fees of ₹ 30 and call rate per minute is ₹ 0.40.

II Package: In this package, company charges a fixed monthly fees of ₹ 40 and call rate per minute is ₹ 0.50.

Consider x as the charges per minute and y as the total amount.

On the basis of the above information, solve the following questions:

I Package

(i) Write the linear equation representing in this package.

(ii) If Anita do a call for 40 min, find the amount paid by Anita.

OR

If Kamla pays a total bill of ₹ 250, then find the number of minutes used by her.

II Package

(iii) If Vimal used 50 minutes talk time, then how much amount paid by Vimal?

Very Short Answer Type Questions

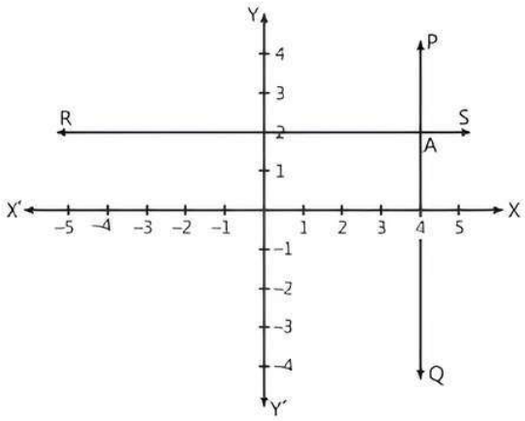
Q 10. **For what value of c, the linear equation $2x + cy = 8$ has y as 4 times of x for its solution?**

Q 11. **Write one solution of $\pi x + 2y = 7$.**

Short Answer Type-I Questions

Q 12. **Let y varies directly as x. When $y = 8$ and $x = 4$, then write a linear equation. Also, determine the value of y when $x = 3$.**

Q 13. **Write the linear equation represented by lines PQ and RS. Also, find the coordinates of intersection of lines PQ and RS.**



Short Answer Type-II Questions

Q 14. If $x = 1$ and $y = 6$ is a solution of the equation $8x - ay + a^2 = 0$, find the value of a.

Q 15. Write the equation $y\sqrt{2} = 7x + \sqrt{5}$ in the form of $ax + by + c = 0$. Check whether $\left(\frac{-\sqrt{5}}{7}, 0\right)$ and $(-1, 3)$ are the solutions of this equation.

Long Answer Type Question

Q 16. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, then express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 2 units. Also, read from the graph, the work done when the distance travelled by the body is (i) 3 units (ii) 4 units.